COVERGENCE OF NUMERICAL SOLUTIONS OF STEP-TYPE WAVEGUIDE DISCONTINUITY PROBLEMS BY MODAL ANALYSIS*

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Abstract

Convergence of modal analysis solutions of step-type waveguide discontinuity problems is studied. The convergence rate depends on the ratio between the number of modal terms retained in different regions. Guidelines for accurate and efficient computations are indicated.

I Introduction

The modal analysis method is a standard computer oriented method for solving discontinuity problems in waveguides. In the formulation the total fields in each guide region are expanded in terms of a complete set of normal modes whose amplitudes are adjusted so as to satisfy the boundary conditions at the discontinuity. This procedure leads directly to a scattering matrix that characterizes the isolated discontinuity. Since the scattering matrix contains information on all the modes either above or below cutoff, a useful wideband equivalent network can be constructed for the analysis of interacting discontinuities in waveguides.

Although the analysis is exact, difficulties can arise in the actual numerical calculations as a result of convergence problems. The numerical solutions use only a finite numer of normal modes, and as a result they may converge to an incorrect value if an improper ratio is chosen between the number of modal terms retained in different regions. This phenomenon, known as 'relative convergence', has been studied for iris-type waveguide discontinuities[1,2].

The class of problems to be considered here is step-type waveguide discontinuity problems. This discontinuity structure is one of the most useful building blocks for waveguide transformers[3] and filters[4,5]. Recently several authors[3,6] have applied the modal analysis method to these problems. However, no discussion has been included on the convergence problem. Due to the increasing interest in developing a wideband equivalent network, it is important to have an understanding of the associated convergence problem and accompanying guidelines for making accurate and efficient computations. This is the motivation for this study.

II Formulation

The problem is a transverse junction between two rectangular waveguides as shown in Fig. 1a. The junction is formed by joining two waveguides of different cross-sections end to end, with concentric axial lines. For arbitrary fields incident from both guides, the total fields in each region are composed of the incident fields and the scattered fields due to the junction. The boundary value problem is solved by first expanding the total fields in terms of the TE and/or TM normal modes and then matching the tangential components of the fields at the junction. The amplitudes of the normal modes are conveniently represented by the elements of column vectors as shown in Fig. 1b, where $(\tilde{a}^{+}, \tilde{a}^{-})$ and $(\tilde{b}^{+}, \tilde{b}^{-})$ are the amplitude vectors of the incident and scattered fields in region I and II, respectively. The dimensions of vectors $\tilde{a}$ and $\tilde{b}$, which indicate the number of normal modes retained, are set to be P and Q, respectively. The continuity condition on the tangential components of the electric and magnetic fields is then applied across the aperture (z = 0). The resulting equations, with the scattered field amplitudes as unknowns, are re-arranged into the form shown in Fig. 1c. The final solution is the scattering matrix, S, containing the four elements given by

$$S_{22} = [Y_2 + H^T Y_1 H]^{-1} [Y_2 - H^T Y_1 H] \quad (1a)$$

$$S_{21} = 2[Y_2 + H^T Y_1 H]^{-1} H^T Y_2 = [I - S_{22}] H^T \quad (1b)$$

$$S_{12} = 2[H Y_2^{-1} H^T Y_1 + I]^{-1} H = H[I + S_{22}] \quad (1c)$$

$$S_{11} = [H Y_2^{-1} H^T Y_1 + I]^{-1} [H Y_2^{-1} H^T Y_1 - I] = -H S_{22} H^T \quad (1d)$$

where I is the identity matrix; $Y_1(P \times P)$ and $Y_2(Q \times Q)$ are diagonal matrices whose diagonal elements are the wave impedances of the corresponding normal modes; $H(P \times Q)$ and its transpose $H^T(Q \times P)$ are the transformation matrices with the element $H_{mn} = H^T_{nm} = \int_{\text{aperture}} \vec{e}_{1m} \cdot \vec{e}_{2n} da$, $\vec{e}_{1n}$ and $\vec{e}_{2n}$ are the transverse electric field components of the n-th normal modes at z=0 for region I and II, respectively.

The scattering parameters S_{11}, S_{21}, S_{12}, and S_{22} are matrices of the same dimension as (PxP), (QxP), (PxQ), and (QxQ), respectively, i.e., they contain not only the scattering characteristics of the fundamental mode, but also those of the higher-order evanescent modes. It is interesting to point out the expressions in eqn.(1) are identical to those obtained in [6] using a conservation of complex power technique. As a result, the solutions of the modal analysis always satisfy the condition of power conservation.

III Numerical Results and Discussion

First, consider a special case of b=d, i.e., two guides are of equal height. This reduces the structure to an H-plane step junction such that any TE_{n0} wave incident excites only TE_{p0} waves with p = 1,2,..... The normal modes and their wave impedances are obtained for both regions and from (1) the scattering parameters are calculated for any TE_{n0} wave.

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*This work was supported by Naval Postgraduate School Foundation Research Program.

In previous publications [7,8] the TE_{10} characteristics of the H-plane step junction have been analyzed with various methods. Although the published data are not exact, they serve as a good reference for the convergence study. As an example, consider the case

$$a = 3c = 0.71\lambda; \quad a_n^+ = \begin{cases} 1, & n = 1 \\ 0, & n > 1 \end{cases};$$

$$b_n^+ = 0 \ , \quad n = 1,2,\ldots$$

which corresponds to an incident TE_{10} wave in the larger guide. Since the narrow guide is below cutoff at the chosen frequency, the normalized input admittance looking from the larger guide is purely susceptive, i.e., $Y_{in} = jB$. In Fig. 2 the susceptance B obtained from (1) is plotted versus P for the fixed ratios (Q/P) = (1/3), (2/3), and (3/3). Note that as P increases all three curves converge to values within 0.5% difference. Furthermore, these values agree with the data provided by Waveguide Handbook [7] within the specified 1% accuracy. It is also observed that the values of B converge with respect to P to the asymptotic value most rapidly when Q/P = c/a.

In Fig. 3 the susceptance B is calculated with a fixed P and a varying Q. It is noticed that B approaches the asymptotic value before Q reaches the value of (c/a)P; beyond that point it drops and converges to a smaller value. However, the error is not very large. From this we conclude that, as long as both P and Q are large, the Q/P ratio does not affect the solution significantly. However, for efficient numerical computations one should keep the ratio Q/P = c/a. This criterion becomes more important for the case of a double-step junction where the numerical computation is much more involved.

For a double-step junction an incident TE wave would excite all the TE_{mn} and TM_{mn} normal modes, or more appropriately the TE_{mn}-to-x normal modes. The total number of normal modes, P, retained in guide I is M1xN1, where M1 and N1 are the maximum values for m and n, respectively. Even for a moderate value of M1 = N1 = 20, P becomes so large (P = 400) that the computation task is difficult and expensive. Therefore, it is essential guidelines be available for use in obtaining rapid convergence.

A number of numerical calculations were made and it was noticed that the convergence behavior was similar to the case of the H-plane step junction. In particular the following points were noted:

(1) The solutions approach the correct values as long as the values of M1, N1, M2, and N2 are large.

(2) Accurate and efficient solutions can be obtained by maintaining the ratios: M2/M1 = c/a; N2/N1 = d/b; N1/M1 = b/a.

This behavior is demonstrated by a typical example given in Fig. 4. Notice that the change of N1/M1 from 1/1 to 1/2 does not affect the convergence rate very much, but it does reduce the computation efforts significantly.

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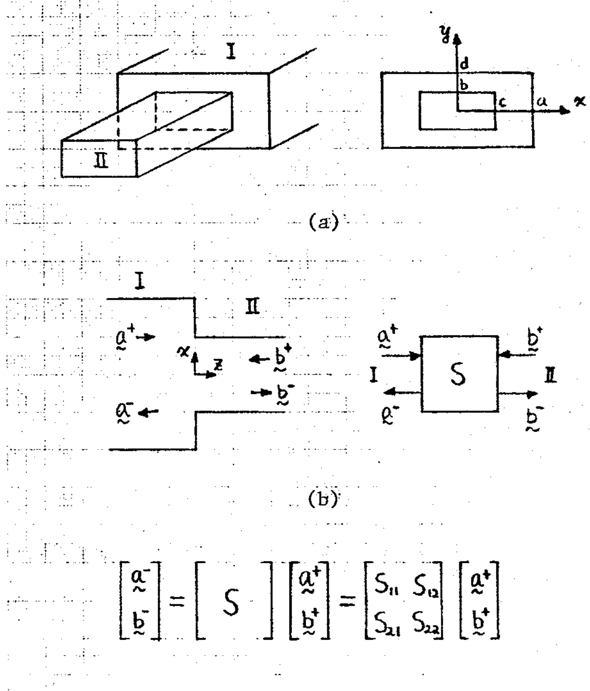

Fig. 1 (a) Geometry and dimensions,
(b) Amplitude vector representation for incident and scattered fields,
(c) Generalized S-parameters.

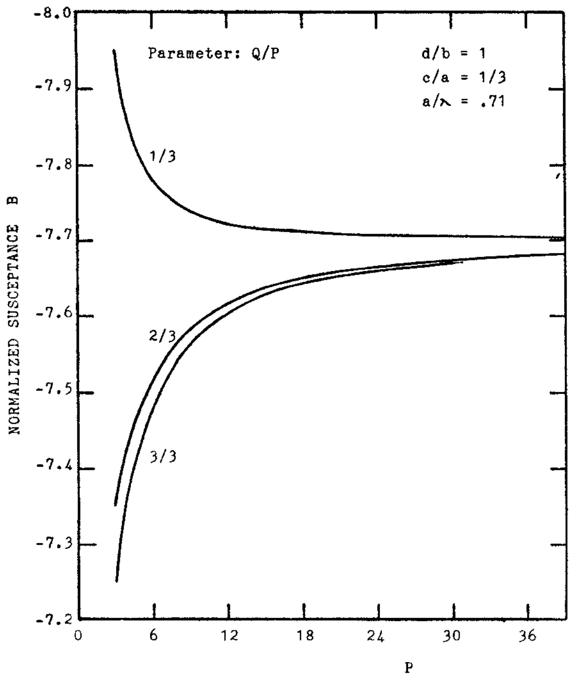

Fig. 2 TE_{10}-mode susceptance of an H-plane step discontinuity between two rectangular waveguides with fixed Q/P.

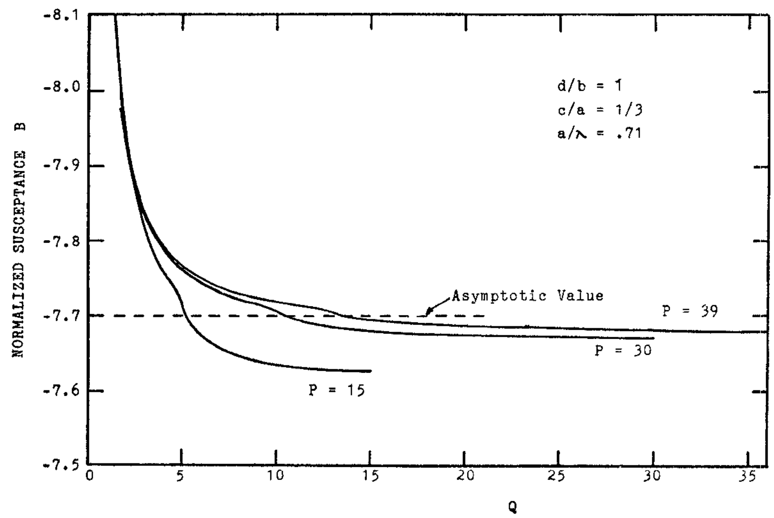

Fig. 3 TE_{10}-mode susceptance of an H-plane step discontinuity between two rectangular waveguides with fixed P.

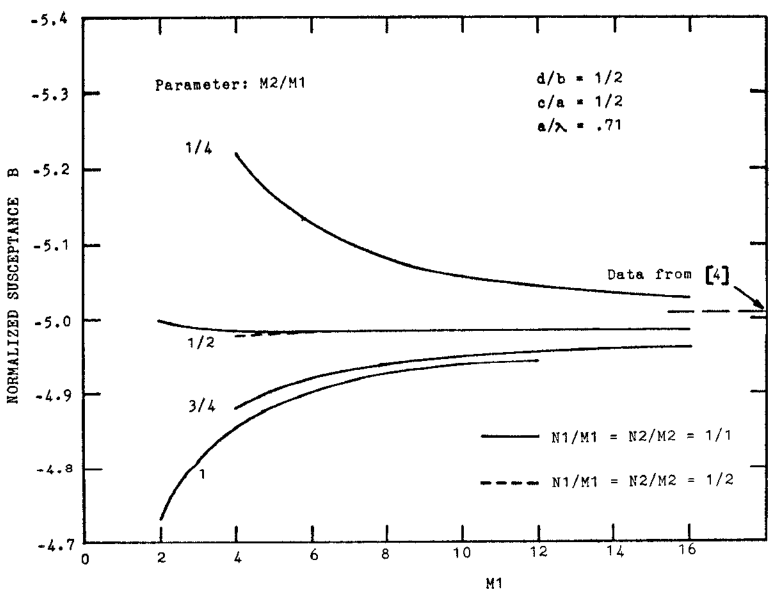

Fig. 4 TE_{10}-mode susceptance of double-step discontinuity between two rectangular waveguides with fixed M2/M1.